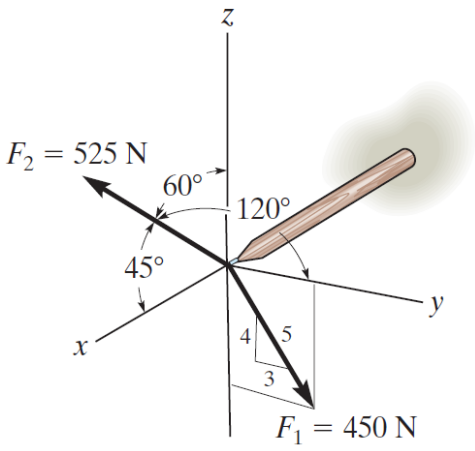
Problem 2-70

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



Prob. 2–70

Solution

Let θ be the angle that $\mathbf{F}_1$ makes with the y-axis.

$$\tan\theta = \frac{4}{3} \quad \rightarrow \quad \theta = \tan^{-1}\left(\frac{4}{3}\right) \approx 53.1^\circ$$

Write each of the forces in component form.

$$\mathbf{F}_1 = 450\,\langle 0, \cos\theta, -\sin\theta\rangle \text{ N} = 450\left\langle 0, \frac{3}{5}, -\frac{4}{5}\right\rangle \text{ N} = \langle 0, 270, -360\rangle \text{ N}$$

$$\mathbf{F}_2 = 525\,\langle \cos 45^\circ, \cos 120^\circ, \cos 60^\circ\rangle \text{ N}$$

Add them together to get the resultant force.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= \langle 525\cos 45^\circ, 270 + 525\cos 120^\circ, -360 + 525\cos 60^\circ\rangle \text{ N} \\ &\approx \langle 371, 7.5, -97.5\rangle \text{ N}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &= \sqrt{(525\cos 45^\circ)^2 + (270 + 525\cos 120^\circ)^2 + (-360 + 525\cos 60^\circ)^2} \text{ N} \\ &\approx 384 \text{ N}\end{aligned}$$

Divide the resultant force by its magnitude to get a unit vector in the same direction.

$$\frac{\mathbf{F}_R}{|\mathbf{F}_R|} \approx \frac{\langle 371, 7.5, -97.5\rangle \text{ N}}{384 \text{ N}}$$

The direction angles for $\mathbf{F}_R$ can now be determined.

$$\begin{cases} \cos\alpha \approx \dfrac{371}{384} \\ \cos\beta \approx \dfrac{7.1}{384} \\ \cos\gamma \approx -\dfrac{97.5}{384} \end{cases} \rightarrow \begin{cases} \alpha \approx 14.8^\circ \\ \beta \approx 88.9^\circ \\ \gamma \approx 105^\circ \end{cases}$$

The resultant force is illustrated below in the coordinate system.